Polynomial and Rational Functions Test Review

We have learned a lot in this unit. This review is not meant to show every possible type of question. Rather, it is meant to jog your memory regarding the topics we've seen and help you realize which topics you need to review more by going back to the homework/notes. If you use AP Classroom or Flipped Math, the relevant topics are 1.1 through 1.11 (except 1.11.C).

1. Let $f(x) = 2x^2 + 5x + 4$ and $g(x) = x^2 + x + 9$. Over what interval(s) is $f(x) \geq g(x)$. Express your answer in interval notation.

2. Let $h(x) = \frac{4x^2-x-3}{x^2-4x+4}$.
   a. Find the equation of each asymptote of $h(x)$ (horizontal, vertical, and slant).
   b. Find the coordinates of each hole in the graph of $h(x)$.
   c. Find the zeros of $h(x)$.
   d. Solve the following inequality $h(x) \geq 0$. Express the solution in interval notation.

3. Let $g(x) = \frac{3(x-1)^2(x+4)}{x^2-1}$.
   a. Find the equation of each vertical asymptote of $g(x)$.
   b. Find the coordinates of each hole in the graph of $g(x)$.
   c. Determine the end behavior of g as x decreases without bound. Express your answer using the mathematical notation of a limit.

4. Let $f(x) = \frac{4-x^2}{x^2-4x+4}$.
   a. Find the equation of each vertical asymptote of $f(x)$.
   b. Find the coordinates of each hole in the graph of $f(x)$.
   c. Find the zeros of $f(x)$.
   d. Determine the end behavior of f as x increases without bound. Express your answer using the mathematical notation of a limit.

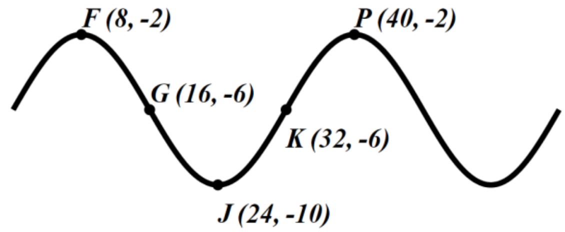
5. The graph of a polynomial function $f(x)$ is tangent to the x-axis at the point $(2,0)$ and crosses the x-axis at the point $(-4,0)$. If $f(2-i) = 0$, what is the least possible degree of $f(x)$?

6. Let $g(x) = (x^2+1)^3(x^2-4)(3x-1)(5x^2-9x-2)$
   a. How many distinct real zeros does $g(x)$ have?
   b. How many distinct non-real zeros does $g(x)$ have?
   c. At how many points is the graph of $y = g(x)$ tangent to the x-axis?
   d. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

7. Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for $f(x) = \frac{(x-1)(3x-2)(x+4)^2}{1-2x}$

8. Find the horizontal asymptote of $f(x) = \frac{-10+x^2}{3x-4x^2+5}$.

9. Consider the graph of $h(t)$ shown below with five labelled points.



a. If the t-coordinate of G is t_1 and the t-coordinate of J is t_2, then on the interval (t_1, t_2), which one of the following is true about h?
   (A) h is positive and increasing
   (B) h is positive and decreasing
   (C) h is negative and increasing
   (D) h is negative and decreasing

b. Let the t-coordinate of G be t_1 and let the t-coordinate of J be t_2. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.
   (A) The graph of h is concave up on the interval (t_1, t_2). The rate of change of h is decreasing on that interval.
   (B) The graph of h is concave up on the interval (t_1, t_2). The rate of change of h is increasing on that interval.
   (C) The graph of h is concave down on the interval (t_1, t_2). The rate of change of h is increasing on that interval.
   (D) The graph of h is concave down on the interval (t_1, t_2). The rate of change of h is decreasing on that interval.

10. Find the domain of each of the following functions. Express your answer in interval notation.
   a. $y = \sqrt{(x-2)^2(x+3)}$
   b. $y = \sqrt[3]{x^4 + 4x^3 - 9x^2 - 36x}$
   c. $y = \sqrt[4]{-2x^4 + 12x^2}$

11. The table below shows values of the function $g(x)$.

x	0	3	6	9	12	15
$g(x)$	2	k	3	8	16	27

   a. Find the average rate of change of $g(x)$ over the interval $[6,15]$
   b. If $g(x)$ is a quadratic function, find the value of k.

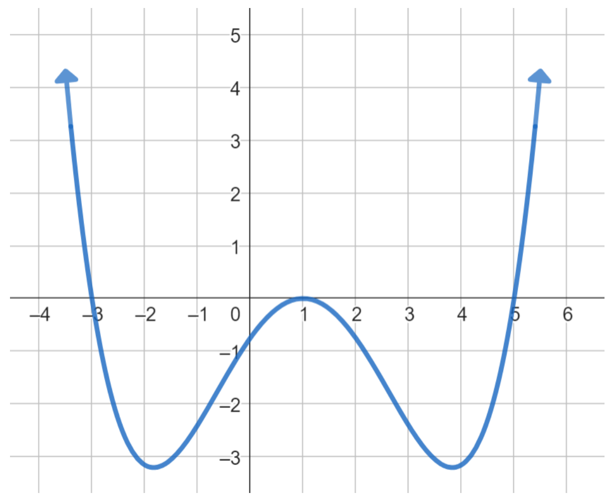
12. For each of the following rational functions, determine the horizontal or slant asymptote.
   a. $y = \frac{2x^2-6x-13}{2x+3}$
   b. $y = \frac{2x^2-3x-1}{x^2-2x+1}$

13. For each of the following, decide if $f(x)$ and $g(x)$ are equivalent. That is, decide if their outputs agree for every possible input.
    a. $f(x) = \frac{2x^2+5x-17}{x+4}$ and $g(x) = 2x - 3 - \frac{5}{x+4}$
    b. $f(x) = \frac{2x^2+x-6}{x^2+6x+9}$ and $g(x) = \frac{2x-5}{x+3}$
    c. $f(x) = \frac{2x^2+x-10}{x^2-5x+6}$ and $g(x) = \frac{2x+5}{x-3}$
    d. $f(x) = \frac{-3x+17}{x^2-2x-3}$ and $g(x) = \frac{2}{x-3} - \frac{5}{x+1}$

Questions #14-17 pertain to the graph of $y = h(x)$ shown below to the right.

14. State the end behavior of $h(x)$ using limit notation.

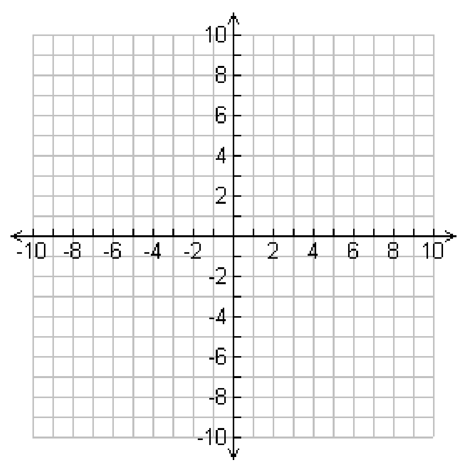
15. At which x-values does $h(x)$ appear to have points of inflection? Select all that apply:
    A) -2.5
    B) -1.5
    C) -0.5
    D) 0.5
    E) 1.5
    F) 2.5
    G) 3.5
    H) 4.5



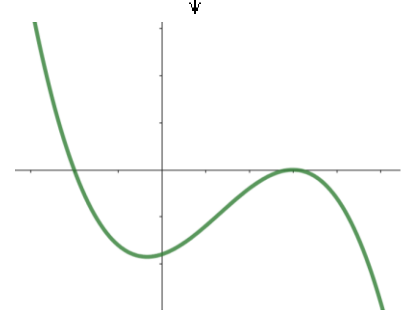
16. How many distinct real zeros does $h(x)$ have?

17. If $h(x)$ is a sixth-degree polynomial, which of the following are possible? Select all that apply.
    A) $h(x)$ has only real zeros
    B) $h(x)$ has one non-real zero
    C) $h(x)$ has two non-real zeros
    D) $h(x)$ has three non-real zeros
    E) $h(x)$ has four non-real zeros

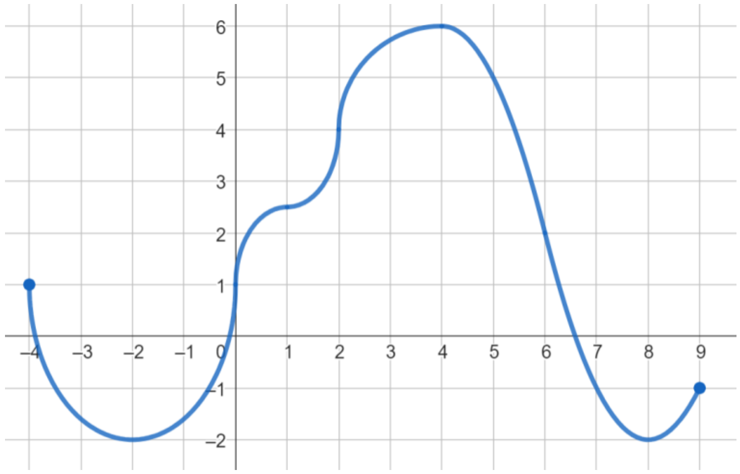
18. On the graph provided to the right, sketch a rational function $g(x)$ for which the following are true:
    - As the input increases without bound, the output of $g(x)$ becomes arbitrarily close to 3.
    - $\lim_{x \to -2^-} g(x) = -\infty$
    - $\lim_{x \to -2^+} g(x) = \infty$
    - $\lim_{x \to -1} g(x) = 4$



19. The graph of $y = f(x)$ is shown to the right. Which one of the following could be the equation of $f(x)$?
    A) $f(x) = -4(x-3)(x+2)^2$
    B) $f(x) = 5(x+2)(x-3)^2$
    C) $f(x) = -3(x+2)(x-3)^2$
    D) $f(x) = 6(x-3)(x+2)^2$



Below is the graph of $f(x)$. Questions #20-26 pertain to this function.



20. Over the interval $(0,1)$, $f(x)$ is...
    A) Increasing and concave down
    B) Decreasing and concave down
    C) Increasing and concave up
    D) Decreasing and concave up

21. At what x-value(s) are the local maxima of $f(x)$?

22. What is the global maximum of $f(x)$?

23. How many relative extrema does $f(x)$ have?

24. Where are the relative minima of $f(x)$?

25. At what x-values does $f(x)$ appear to have points of inflection?

26. What is the absolute minimum of $f(x)$?

Each of the following questions (#27-#32) describes a polynomial function $f(x)$. However, each scenario is impossible. Give a reason why the proposed polynomial is impossible.

27. $f(-2) = 10$, $f(0) = 8$, $f(2) = 3$, f is concave up over $(-\infty, \infty)$
28. The leading coefficient of $f(x)$ is 5, $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = -\infty$, $f(x)$ has no global minimum
29. $y = f(x)$ has four x-intercepts, $f(x)$ has one local maximum, $f(x)$ has one local minimum, $\lim_{x\to\infty} f(x) = \infty$
30. $f(3) = -4$ is a relative maximum, $f(-3) = 4$ is a relative minimum, f is an even function, f has no non-real zeros.
31. -3 is a zero of f with multiplicity 2, f is of degree 3, f is an odd function, $\lim_{x\to\infty} f(x) = \infty$
32. The graph of $f(x)$ is tangent to the x-axis at $x = 3$, f is an even function, $2 - i$ is a zero of f, f has degree 4

Each of the following questions (#33-36) describes a rational function $f(x)$ that is of the form $f(x) = \frac{n(x)}{d(x)}$ where $n(x)$ and $d(x)$ are polynomial functions. However, each scenario is impossible. Give a reason why the proposed rational function is impossible.

33. The degree of $n(x)$ and $d(x)$ are equal, $n(x)$ has only non-real zeros, $d(x)$ has only real zeros, $f(x)$ has no vertical asymptotes.
34. The degree of $n(x)$ and $d(x)$ are equal, $n(x)$ has only real zeros, $d(x)$ has exactly two non-real zeros, $\lim_{x\to\infty} f(x) = -\infty$
35. $n(x)$ is an even function, $d(x)$ is an odd function $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 4$
36. The domain of $f(x)$ is all real numbers, $d(x)$ is a cubic polynomial, $\lim_{x\to\infty} n(x) = \infty$, $n(x)$ has no real zeros

37. Let $f(x) = ax^b - 5x^3 + 2x - 11$. Given that $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$, which one of the following choices presents possible values of a and b?

A) $a = 3$ and $b = 2$

B) $a = -3$ and $b = 2$

C) $a = 3$ and $b = 4$

D) $a = -3$ and $b = 4$

E) $a = 3$ and $b = 5$

F) $a = -3$ and $b = 5$

38. If $k(x)$ is an odd function and $k(2) = 5$, which one of the following must be true?

A) $k(-2) = -5$

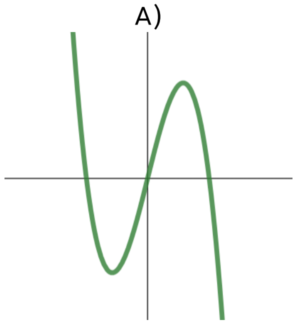
B) $k(-2) = 5$

C) $k(5) = 2$

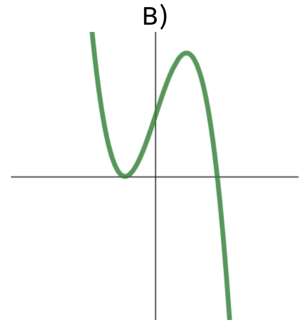
D) $k(5) = -2$

39. Which one of the following graphs could be a cubic function for which $3 - 5i$ is a zero?

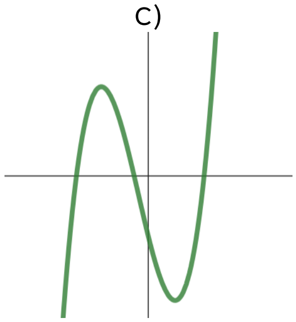
A)



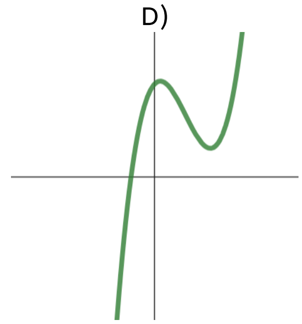
B)



C)



D)



40. Let $f(x)$ be an even function. If $f(x)$ is increasing and concave down on the interval $(1,5)$, then what can be said of $f(x)$ on the interval $(-5,-1)$? Select one of the following.

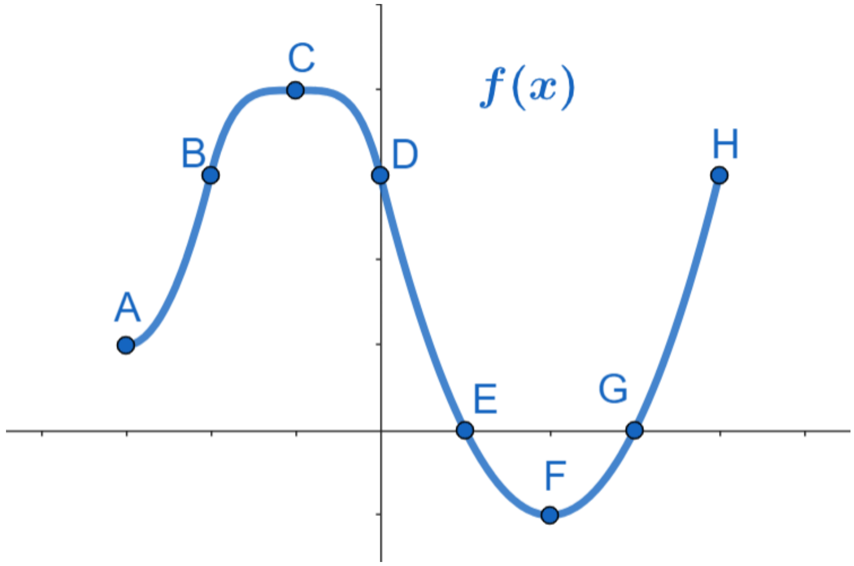
A) $f(x)$ is increasing and concave up over the interval $(-5,-1)$

B) $f(x)$ is decreasing and concave up over the interval $(-5,-1)$

C) $f(x)$ is increasing and concave down over the interval $(-5,-1)$

D) $f(x)$ is decreasing and concave down over the interval $(-5,-1)$

Use the below graph of $f(x)$ to answer all questions on this page.



41. Over which one of the following intervals is the rate of change of $f(x)$ positive and decreasing?
    A) (B, C)
    B) (C, D)
    C) (D, E)
    D) (F, G)

42. Over which one of the following intervals is $f(x)$ decreasing at a decreasing rate?
    A) (A, B)
    B) (C, D)
    C) (E, F)
    D) (G, H)

43. Over which one of the following intervals is the average rate of change of $f(x)$ zero?
    A) (A, C)
    B) (B, D)
    C) (C, E)
    D) (D, F)

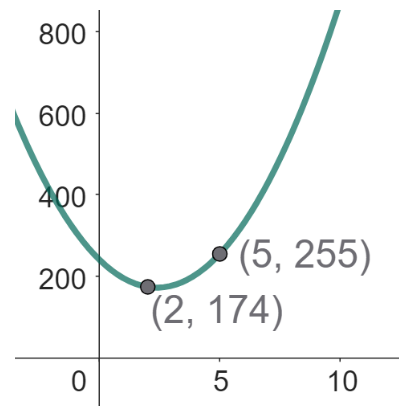
44. At which one of the following x-values is the rate of change of $f(x)$ greatest?
    A) C
    B) D
    C) G
    D) H

45. Over what interval(s) is $f(x) > 0$?

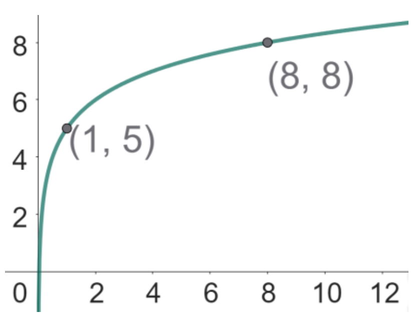
46. How many points of inflection does $f(x)$ have?

47. For a certain chemical gas reaction, the speed of the reaction changes over time. One second into the reaction ($t = 1$), the speed is $5\frac{\text{L}}{\text{s}}$. Thirteen seconds into the reaction ($t = 13$), the speed is $11\frac{\text{L}}{\text{s}}$. Throughout the first minute of the reaction, the reaction speed can be modelled by $R(t) = a + b\sqrt{t+3}$, where $R(t)$ is the speed t seconds after the reaction began.
   A) Use the given data to write two equations that can be used to find the values of the constants a and b in the expression for $R(t)$.
   B)
      (i) Use the given data to find the average rate of change of the reaction speed, in $\frac{\text{L}}{\text{s}}$ per second, from $t = 1$ to $t = 13$ seconds. Express your answer as a decimal approximation. Show the computations that lead to your answer.
      (ii) Use the average rate of change found in (i) to estimate the reaction speed at time $t = 21$ seconds. Show the work that leads to your answer.

To the right is the graph of $y = f(t)$. $f(2) = 174$ and $f(5) = 255$.
The average rate of change of this function over the interval $2 \le t \le 5$ is 27.



48. If this average rate of change is used to approximate $f(7)$, would it result in an overestimate or underestimate of the actual value of $f(7)$?
49. If this average rate of change is used to approximate $f(3)$, would it result in an overestimate or underestimate of the actual value of $f(3)$?
50. If this average rate of change is used to approximate $f(1)$, would it result in an overestimate or underestimate of the actual value of $f(1)$?



51. To the left is the graph of $y = g(t)$. The average rate of change over the interval $1 \le t \le 8$ may be used to approximate $g(k)$ for $k > 8$. Which of the following statements **best** explains why these approximations would overestimate the actual value for $g(k)$?
(A) $g(t)$ is a concave down function. Beyond a certain point, the average rate of change increases faster than $g(t)$, so the average rate of change has a higher output than $g(t)$.
(B) $g(t)$ is a concave down function. Estimates based on average rate of change are outputs of the secant line passing through g at $t = 1$ and $t = 8$. Since $g(t)$ is concave down, the graph of the secant line is above the graph of $g(t)$ for values of t greater than 8.
(C) $g(t)$ is a concave down function. Since g is concave down, the rate of change of g decreases. For $t > 8$, the rate of change has decreased enough that the average rate of change is greater than the output of $g(t)$. We can see this visually by noticing that the average rate of change is above the graph of $g(t)$.
(D) $g(t)$ is a concave down function. We are comparing the actual value of $g(t)$ to outputs of a secant line passing through $g(t)$. Since the secant line is above the graph of $y = g(t)$, we know estimates based on the average rate of change are greater (and therefore overestimate) than the actual value of $g(t)$.

The wording of #3c, 4d, and 6d is modelled after a part of a question that will definitely appear on the AP exam. On the test, you can expect the same wording. **That question/those questions will be free response.** You need to know how to write the limit properly. That being said, if you'd like to practice similar questions in a multiple choice format, the link to the right can help.

#9 is modelled after a part of a question that will definitely appear on the AP exam. On the test, you can expect a similar problem. If you'd like more practice on this, the link to the right can help.

On the Day 2 homework, there was a QR code/link to a game that helped practice the difference between similar sounding words. It practiced what it means to call $f(x)$ increasing/decreasing/concave up/concave down versus what it means to call a rate of change positive/negative/increasing/decreasing. Because we had not yet discussed inequalities, it never presented questions where we had to label $f(x)$ as positive or negative. But this is something you should practice. Below to the left is the QR code/link from the Day 2 homework. Below to the right is a QR code/link that is a step more difficult because it adds in questions about $f(x)$ being positive/negative.

We learned some Desmos skills over the past couple classes. On the test, you cannot use a calculator/Desmos. So a lot of the material from the past couple classes could not be asked on the test. Questions like #47(A) do not need Desmos, so it is fair game. The question that usually follows that ("find a and b") will not be on the test. If a question like #47(B) appears, it will involve numbers that we can reasonably expect you to calculate by hand.

Polynomial and Rational Functions Test Review Solutions

1.
$$2x^2+5x+4 \ge x^2+x+9$$
$$x^2+4x-5 \ge 0$$
$$(x+5)(x-1) \ge 0$$

Answer: $(-\infty, -\mathbf{5}] \cup [\mathbf{1}, \infty)$

2. $h(x) = \frac{4x^2-x-3}{x^2-4x+4} = \frac{(x-1)(4x+3)}{(x-2)(x-2)}$
   a. Horizontal Asymptote: $y = 4$
      Vertical Asymptote: $x = 2$
      Slant Asymptote: None
   b. No Holes
   c. $x = 1$ and $x = -\frac{3}{4}$
   d. $\left(-\infty, -\frac{3}{4}\right] \cup [1,2) \cup (2, \infty)$
3. $g(x) = \frac{3(x-1)^2(x+4)}{x^2-1} = \frac{3(x-1)^2(x+4)}{(x-1)(x+1)} = \frac{3(x-1)(x+4)}{x+1}$ for $x \neq 1$
   a. $x = -1$
   b. Hole: $(1,0)$
   c. $\lim_{x\to-\infty} g(x) = -\infty$
4. $f(x) = \frac{4-x^2}{x^2-4x+4} = \frac{-(x-2)(x+2)}{(x-2)(x-2)} = -\frac{x+2}{x-2}$
   a. VA: $x = 2$
   b. Hole: None
   c. $x = -2$
   d. $\lim_{x\to\infty} f(x) = -1$
5. 5 ($x = 2 - i$ is a zero, $x = 2 + i$ is a zero, $x = -4$ is a zero with odd multiplicity and $x = 2$ is a zero with even multiplicity)e
6. $g(x) = (x^2+1)^3(x^2-4)(3x-1)(5x^2-9x-2) = (x^2+1)^3(x-2)(x+2)(3x-1)(x-2)(5x+1) = (x^2+1)^3(x-2)^2(x+2)(3x-1)(5x+1)$
   a. 4 ($x = 2, -2, \frac{1}{3}, -\frac{1}{5}$)
   b. 2 ($x = i$ and $x = -i$)
   c. 1 (at the point $(2,0)$)
   d. $\lim_{x\to\infty} g(x) = \infty$
7. $\lim_{x\to\infty} f(x) = -\infty$ and $\lim_{x\to-\infty} f(x) = \infty$ (Because we only need to analyze $\frac{3x^4}{-2x} = -\frac{3}{2}x^3$)
8. $y = -\frac{1}{4}$
9.
   a. (D). This question is talking about the function itself, not the rate of change. The function is negative because the y-values of the points between G and J are negative. The function is decreasing because from G to J, the y-values went down.
   b. (B). This question is about the rate of change. The graph of h is concave up between point G and point J. A different way to say that is “The rate of change is increasing” between those points. Option C is true but irrelevant to the question.
10.
   a.
   $$(x-2)^2(x+3) \ge 0$$
   Answer: $[-\mathbf{3}, \infty)$
   b. $(-\infty, \infty)$

c.

$$-2x^4 + 12x^2 \geq 0$$
$$(-2x^2)(x^2 - 6) \geq 0$$

Answer: $\left[-\sqrt{6}, \sqrt{6}\right]$

11.
   a. $\frac{g(15)-g(6)}{(15)-(6)} = \frac{27-3}{15-6} = \frac{24}{9} = \frac{8}{3}$
   b. 1

12.
   a. Slant Asymptote: $y = x - \frac{9}{2}$
   b. Horizontal Asymptote: $y = 2$

13.
   a. Yes, $f(x) = g(x)$
   b. No
   c. No. $g(2) = -7$ but $f(2)$ is undefined. So, these two functions are not equivalent.
   d. Yes, $f(x) = g(x)$

14. $\lim_{x\to\infty} h(x) = \infty$ and $\lim_{x\to-\infty} h(x) = \infty$
15. C and F (-0.5 and 2.5)
16. 3 DISTINCT zeros
17. A and C

    A is possible if the zeros of $h(x)$ are $x = -3$ (mult. 1), $x = 1$ (mult. 4), and $x = 5$ (mult. 1)

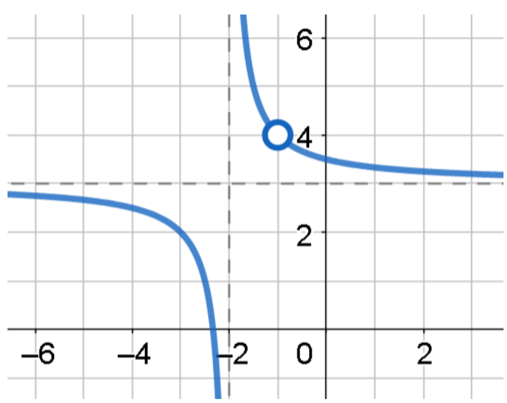
    B is impossible. Non-real zeros come in in conjugate pairs. Impossible to have only one.

    C is possible if the zeros of $h(x)$ are $x = -3$ (mult. 1), $x = 1$ (mult. 2), and $x = 5$ (mult. 1) and two non-real

    D is impossible. Non-real zeros come in conjugate pairs. Impossible to have an odd number of non-real zeros.

    E is impossible. There are at least four real zeros. That means there are at the most two non-real zeros.

18.



19. C
20. A
21. $x = -4$, $x = 4$, $x = 9$
22. 6 (Writing $f(4) = 6$ is fine. Writing $(4,6)$ is not)
23. 5
24. $x = -2$ and $x = 8$
25. $x = 0$, $x = 1$, $x = 2$, and $x = 6$
26. -2 (Writing $f(-2) = -2$ is fine. Writing $(-2,-2)$ is not)

27. The average rate of change over the interval $-2 \leq x \leq 0$ is $\frac{f(0)-f(-2)}{(0)-(-2)} = \frac{-2}{2} = -1$

    The average rate of change over the interval $0 \leq x \leq 2$ is $\frac{f(2)-f(0)}{2-0} = \frac{-5}{2} = -2.5$

    The rate of change decreased. If the function were concave up, the rate of change would have increased.
28. A polynomial with a positive leading coefficient would have $\lim_{x\to\infty} f(x) = \infty$
29. Between every pair of x-intercepts of a polynomial is a relative extremum. With four x-intercepts, there must be at least three relative extrema. So, it is impossible that there are only two (one rel max and one rel min)
30. For an even function, $f(3)$ and $f(-3)$ should be equal
31. If $x = -3$ is a zero of multiplicity 2, then $(-3,0)$ should be a "bouncing" x-intercept.
    Because the function is odd, it has origin symmetry. So $(3,0)$ is also going to be "bouncing" (though it will bounce off the other side of the x-axis)
    So $x = 3$ must also be a zero of even multiplicity.
    $f(x)$ therefore has at least 4 zeroz.
    But f is stated to be degree 3!
32. If the graph is tangent to the x-axis at $(3,0)$ then $x = 3$ is a zero of even multiplicity (at least 2)
    If the function is even, $x = -3$ must also be a zero of even multiplicity (at least 2)
    $2 - i$ being a zero implies $2 + i$ is also a zero.
    So, there are at least 6 zeros.
    But f is stated to be degree 4.
33. If the denominator has only real zeros and the numerator has only non-real zeros, there will be no factors in common that will cancel. So, all of the zeros of the denominator will correspond with vertical asymptotes. This contradicts the statement that $f(x)$ has no vertical asymptotes.
34. If the degree of $n(x)$ and $d(x)$ are the same, there will be a horizontal asymptote. This contradicts $\lim_{x\to\infty} f(x) = -\infty$
35. $\lim_{x\to\infty} f(x) = 4$ implies there is a horizontal asymptote $y = 4$. Rational functions approach their horizontal asymptotes on both sides. So, $\lim_{x\to-\infty} f(x) = 4$ as well. But this contradicts the limit given in the third statement.
36. If $d(x)$ is cubic, it has at least one real zero. At that value where the denominator is zero, $f(x)$ will be undefined. It is impossible that $f(x)$ has a domain of all real numbers.
37. D
38. A
39. D (cubic means three zeros. If $3 - 5i$ is a zero, so is $3 + 5i$. So only one real zero)
40. D
41. A
    "Rate of Change Positive" = "$f(x)$ increasing" and "Rate of Change Decreasing" = "$f(x)$ concave down"
42. B
    "Decreasing rate" = "Concave down function"
43. B
    "Average rate of change" = "Slope between two points"
44. H
    At point C, the rate of change is zero. At point D the rate of change is negative. At points G and H the rate of change is positive, so one of those corresponds with the greatest rate of change. Visually we may spot that the graph is steeper than H, but even if we don't concavity is a hint: The graph is concave up everywhere after point D. So, the rate of change increases from D to E and from E to F and from F to G and from G to H. If the rate of change increased from G to H, certainly H has the higher rate of change.
45. $[A, E) \cup (G, H]$
    $f(x) > 0$ visually looks like points above the x-axis

46. 2 (point B and point D are the POIs)
47.
    A) $5 = a + b\sqrt{1+3}$ and $11 = a + b\sqrt{13+3}$
    B)
        (i) $\frac{R(13)-R(1)}{13-1} = \frac{11-5}{13-1} = \frac{6}{12} = 0.5\frac{\text{L}}{\text{s}}$ per second
        (if no work shown, zero credit. If left as a fraction and not a decimal, zero credit.)
        (ii) $\frac{R(21)-11}{21-13} \approx 0.5$
        $\frac{R(21)-11}{8} \approx 0.5$
        $R(21) - 11 \approx 4$
        $R(21) \approx 15$
        (You may have done this questions differently. That is okay as long as you showed the work. If you are not sure if your approach is valid, ask. I used an "approximately equal" symbol. You do not have to.)
48. Underestimate. This question did not ask for you to share your reasoning, but the reasoning is: $f(t)$ is concave up. Because f is concave up, the secant line passing through f at $t = 2$ and $t = 5$ is below the graph of f for values of t greater than 5. So at $t = 7$, the output of the secant line is less than the actual value of f.
49. Overestimate. This question did not ask for you to share your reasoning, but the reasoning is: $f(t)$ is concave up. Because f is concave up, the secant line passing through f at $t = 2$ and $t = 5$ is above the graph of f for values of t between $t = 2$ and $t = 5$. So at $t = 3$, the output of the secant line is greater than the actual value of f.
50. Underestimate. This question did not ask for you to share your reasoning, but the reasoning is: $f(t)$ is concave up. Because f is concave up, the secant line passing through f at $t = 2$ and $t = 5$ is below the graph of f for values of t less than 2. So at $t = 1$, the output of the secant line is less than the actual value of f.
51. B
    Option A incorrectly uses the phrase "the average rate of change increases". The average rate of change is not increasing. Someone who writes that phrase probably meant "estimates based on the average rate of change are increasing", but that's not the same. Beyond this, the wording is vague ("beyond a certain point"...what point?) and needs to mention t-values to be really solid.
    Option C incorrectly uses the phrase "the average rate of change is greater than the output of $g(t)$". The average rate of change is $\frac{8-5}{8-1} = \frac{3}{7} = 0.429$. That certainly isn't greater than the y-values on the graph for $t > 8$. And even if it were, that isn't relevant. Someone who writes "the average rate of change is greater" probably meant "the estimates based on the average rate of change are greater." This is why we use the phrase "secant line", to avoid accidentally saying things like this.
    Option D needs t-values. It claims "the secant line is above the graph of $y = g(t)$". But the secant line isn't always above.